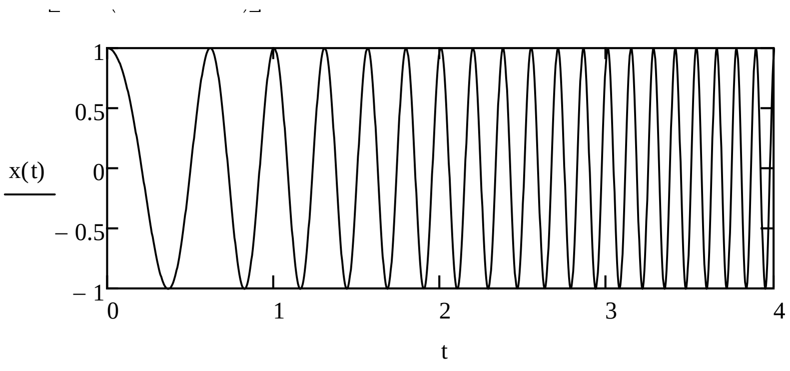
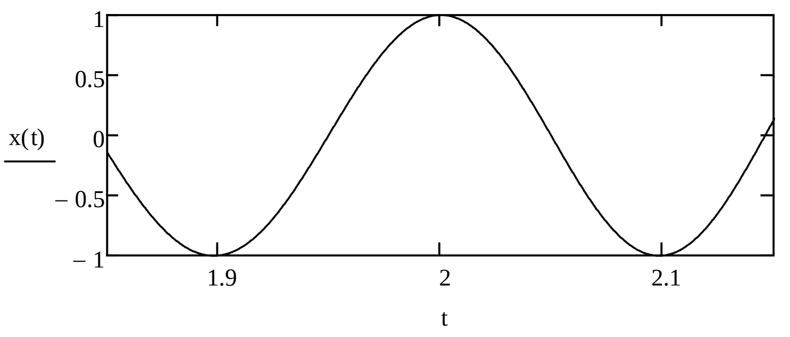
Name_______________________________________ ID_________________________

b. (3 pt) Find $y(t)$ (the output of the LPF).

8. (12 pt) The signal $x(t) = \cos\left(2\pi\left(t^2 + at + 3\right)\right)$ is plotted below. The time unit is in seconds.





Name____ ID________________________

a. (2 pt) Find the instantaneous frequency (in Hz) at $t = 2$. Hint: it is an integer.

1 cycle takes 0.2 sec. So, $f = \frac{1}{0.2} = 5$ Hz

b. (5 pt) Find the value of a. Hint: it is an integer.

$f(t) = \frac{d}{dt}(t^2 + at + 3) = 2t + a.$

$f(2) = 2 \times 2 + a = 5$ $\qquad a = 1.$

c. (5 pt) Find the instantaneous frequency (in Hz) at $t = 5$. Hint: it is an integer.

$f(t) = 2t + a = 2t + 1$

$= 2 \times 5 + 1$

$= 11$ Hz

9. (10 pt) A modulated signal with carrier frequency $f_c = 10^5$ Hz is described by the equation

$$x(t) = 10\cos\left(2\pi f_c t + 5\sin(3000t)\right).$$

a. (5 pt) Find the instantaneous frequency $f(t)$ of $x(t)$.

$\theta(t) = 2\pi f_c t + 5\sin(3000t)$

$f(t) = \frac{1}{2\pi}\theta'(t) = f_c + \underbrace{\frac{5 \times 3000}{2\pi}}_{\Delta f}\cos(3000t)$

$= 10^5 + \frac{7{,}500}{\pi}\cos(3{,}000t)$

From Euler's formula,

$\frac{d}{dx}\sin x = \frac{d}{dx}\left(\frac{1}{2j}(e^{jx} - e^{-jx})\right)$

$= \frac{1}{2j}\left(je^{jx} + je^{-jx}\right)$

$= \frac{1}{2}\left(e^{jx} + e^{-jx}\right) = \cos x$

b. (5 pt) Estimate the bandwidth of $x(t)$ via Carson's rule.

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10. (8 pt) A modulated signal with carrier frequency $f_c = 10^5$ Hz is described by the equation

$$x(t) = 10\cos\left(2\pi f_c t + 5\sin(3000t) + 10\sin(2000\pi t)\right).$$

a. (5 pt) Find the instantaneous frequency $f(t)$ of $x(t)$.

$f(t) = \frac{1}{2\pi}\theta'(t)$

$= f_c + \frac{5\times 3000}{2\pi}\cos(3{,}000t) + \frac{10\times 2000\pi}{2\pi}\sin(2000\pi t)$

$= 10^5 + \frac{7{,}500}{\pi}\cos(3{,}000t) + 10{,}000\sin(2000\pi t)$

b. (3 pt) Estimate the bandwidth of $x(t)$ via Carson's rule.

11. (16 pt) Determine the Nyquist sampling rate and for the signals in the table below. No explanation is needed.

	Nyquist sampling rate	Nyquist sampling interval
$\text{sinc}(200\pi t)$		
$\text{sinc}^2(200\pi t)$		
$\text{sinc}(200\pi t) + 5\text{sinc}^2(120\pi t)$		
$\text{sinc}(100\pi t)\text{sinc}(200\pi t)$		